Problem 5.1: Determine if the systems specified by the following input-output relationships are causal and memoryless. Assume the input is x and output is y.

(a) $y[n] = \frac{1}{2}(x[n] + x[-n])$

(b) $y[n] = (n+1)x[n] + 1$

(c) $y(t) = x(\cos(t))$

(d) $y(t) = x^2(t-1)$

Problem 5.2: Are the systems described in Problem 5.1 stable? Invertible? You may use either a qualitative or quantitative argument to support your reponse.

Problem 5.3: As demonstrated in lecture, the inverse of the accumulator system is the first-difference system. Both systems are shown cascaded in Figure 1, with intermediate signal $w[n]$. Let the input to the arrangement be $x[n] = 2u[n-1] - 2u[n-6]$.

(a) Make a stem plot of $x[n]$, $w[n]$, and $y[n]$. Use a straightedge and label all axes and important features. Show the origin in each plot for context.

(b) Switch the order of the two systems in Figure 1 and repeat (a).

Problem 5.4: Useful in financial analysis, a five-day moving average filter is a discrete-time system whose output today $y[n]$ is the average of the inputs $x[n]$ from the previous five days, not including today. (Each time step n is a day.)

(a) Write the input-output relationship for the five-day moving average filter. Use sigma notion to express your answer compactly.

(b) Is the five-day moving average filter

   (I.) memoryless?

   (II.) causal?

   (III.) stable?

(c) If the input to the the five-day moving average filter is $x[n] = \delta[n]$, use a straightedge to plot the output $y[n]$. Label the axes and all important features. Show the origin for context.

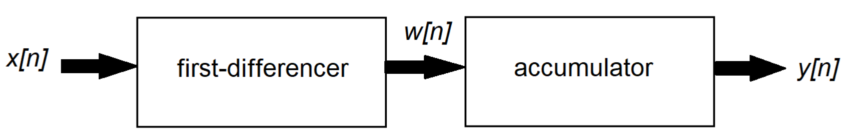

Figure 1

Optional, but testable, problems: From the textbook, Problems 1.16, 1.18(c), 1.27 (omit linearity and time-invariance), 1.28 (omit linearity and time-invariance)