Problem 19.1: The differential equation $\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = x(t)$ models a certain causal LTI system with input $x(t)$ and output $y(t)$. Find the frequency response and the impulse response of the system.

Problem 19.2: The LTI system shown in Figure 1 is composed of three separate LTI systems whose frequency responses are respectively $H_1(j\omega) = \frac{1}{1+j\omega}$, $H_2(j\omega) = \frac{1}{1-j\omega}$, and $H_3(j\omega) = \frac{2}{1+\omega^2}$. Determine the overall system's frequency response.

Problem 19.3*: The suspension system of a car with mass 10^6 kg can be modeled as in Figure 2. The spring system has a spring constant 25 N/m, and the shock absorbers have a damping coefficient $b > 0$ (in kg/s). The input to the road-car system $x(t)$ is the height of the road relative to some reference point, and the output $y(t)$ is the height of the car relative to its original position.

The spring exerts a vertical force on the car that is proportional to the car's vertical displacement, and the shock absorbers exert a vertical force on the car proportional to the car's vertical velocity. According to Newton's laws, the vertical displacement of the car $y(t)$ then is related to the height of the road $x(t)$ by

$$25x(t) - 25y(t) + b\frac{dx(t)}{dt} - b\frac{dy(t)}{dt} = 10^6\frac{d^2y(t)}{dt^2}.$$

(a) Find the frequency response of the causal LTI system.

(b) An input to the system that would approximate an impulse is a speedbump. (A quick "kick" to the shocks.) Show that when the shock absorbers are tuned so $b = 10^4$ kg/s, the car will not oscillate when driving over a speedbump. (Recall a signal that does not oscillate contains no complex exponentials.) *Hint: Quadratic formula.*

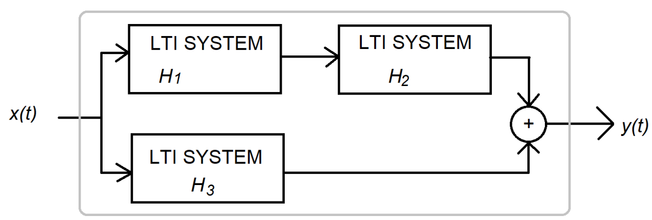

Figure 1

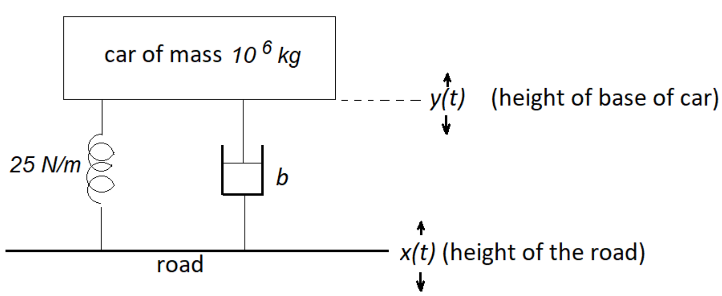

Figure 2

Optional, but testable problems: 4.19, 4.33, 4.34